

Southridge First School

Calculation Strategies

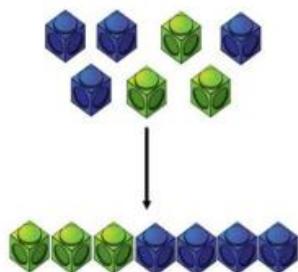


Addition - Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

Year 1

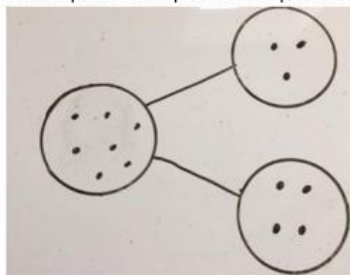
Concrete

Combining two parts to make a whole (using resources e.g., eggs, shells, teddy bears, cubes)



Pictorial

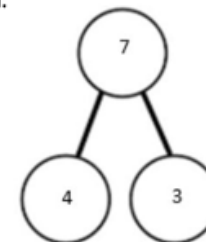
Children represent the cubes using dots or crosses. They could put each part on a part whole model too.



Abstract

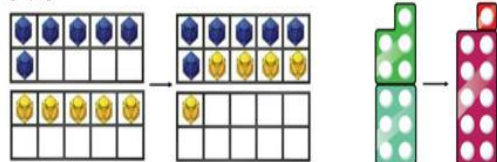
$$4 + 3 = 7$$

Four is a part, 3 is a part and the whole is seven.

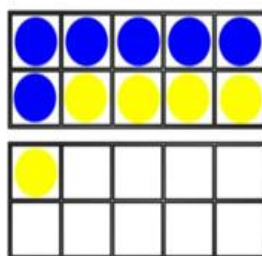


Regrouping to make 10; using ten frames and counters/cubes or using Numicon.

$$6 + 5$$



Children to draw the ten frame and counters/cubes.



Children to develop an understanding of equality e.g.

$$6 + \square = 11$$

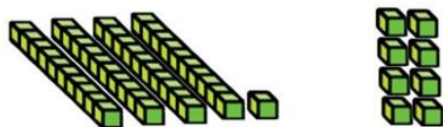
$$6 + 5 = 5 + \square$$

$$6 + 5 = \square + 4$$

Year 2

Concrete

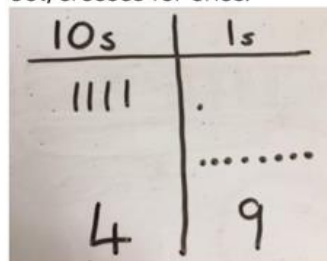
TO + O using base 10. Continue to develop understanding of partitioning and place value.
41 + 8



(Year 1 use the above to add within 20.)

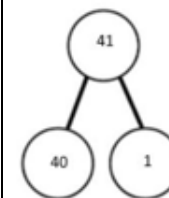
Pictorial

Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.



Abstract

$$41 + 8$$



$$1 + 8 = 9$$

$$40 + 9 = 49$$

Adding multiples of 10. Model using dienes.

$$50 = 30 + 20$$



$$3 \text{ tens} + 5 \text{ tens} = \underline{\quad} \text{ tens}$$

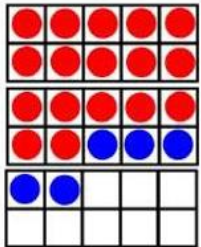
$$30 + 50 = \underline{\quad}$$

Use representations for base ten. Make links to patterns on a hundred square.

$$20 + 30 = 50$$

$$70 = 50 + 20$$

$$40 + \square = 60$$



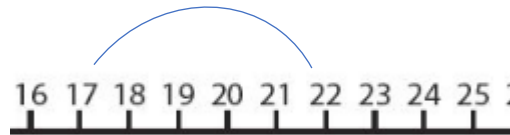
$$17 + 5 = 22$$

Use ten frame to make 'magic ten'.

Children explore the pattern.

$$17 + 5 = 22$$

$$27 + 5 = 32$$



$$17 + 5 = 22$$

$$17 + 5 = 22$$

Explore related facts

$$17 + 5 = 22$$

$$5 + 17 = 22$$

$$22 - 17 = 5$$

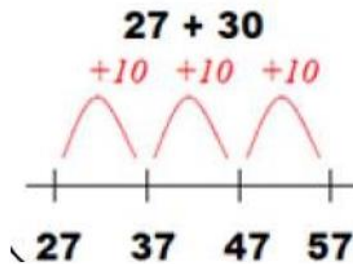
$$22 - 5 = 17$$

22	
17	5



$$25 + 10 = 35$$

Explore that the ones digit does not change.



Explore using hundred squares.

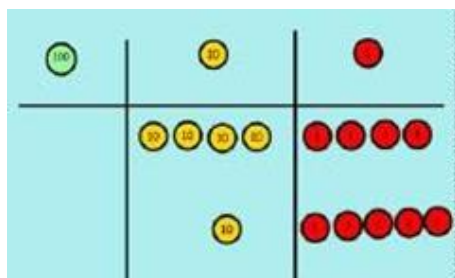
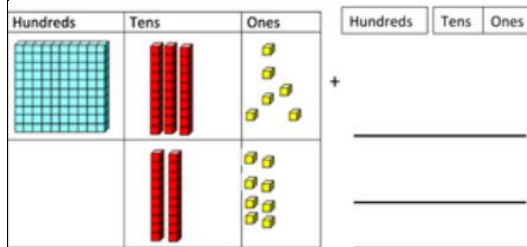
$$27 + 10 = 37$$

$$27 + 20 = 47$$

$$27 + \square = 57$$

Year 3

Concrete



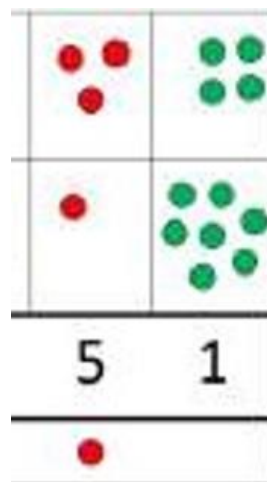
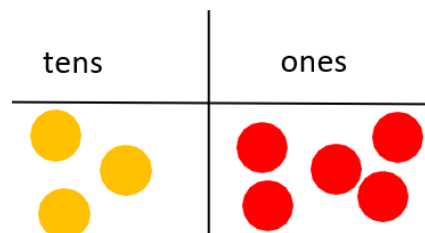
Model column addition – no regrouping with dienes or place value counters.

Add 2- or 3-digit numbers.

Repeat modelling process with regrouping.

Pictorial

Children move to drawing the counters using a tens and one frame.



Children can draw a representation of the grid to further support their understanding, carrying the ten **underneath** the line.

Abstract

$$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$$

Add the ones first, then the tens, then the hundreds.

$$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ 60 + 13 = 73 \end{array}$$

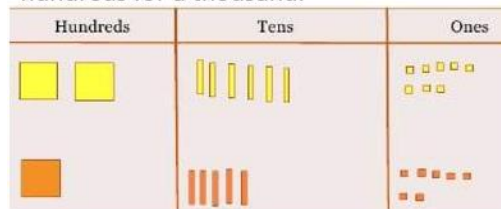
Start by partitioning the numbers before formal column to show the exchange.

$$\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$$

Year 4

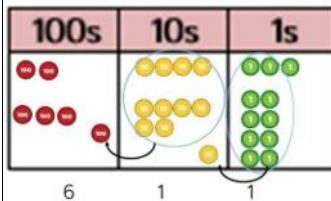
Concrete

Children continue to use dienes or pv counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand.

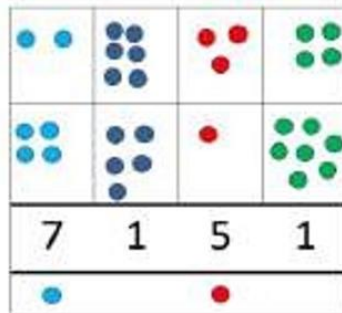


Children add numbers up to 4 digits.

Use of place value counters to add HTO + TO, HTO + HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred.

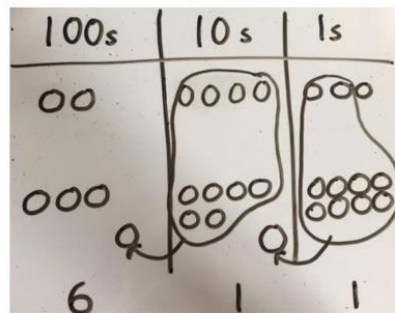


Pictorial

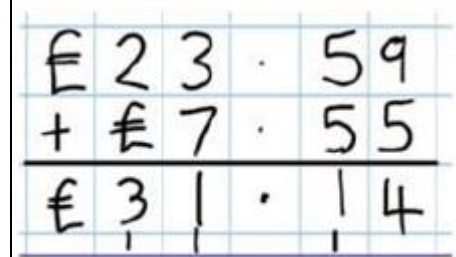
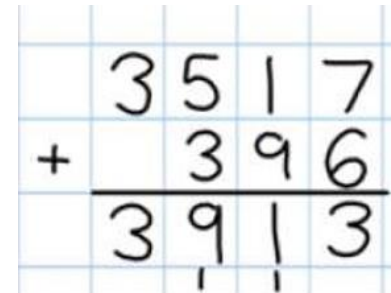


Draw representations using a place value grid.

Children to represent the counters in a place value chart, circling when they make an exchange.

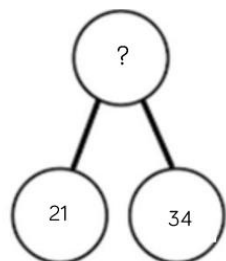


Abstract



Relate process to money and measures.

Conceptual variation: different ways to ask children to solve $21 + 34$



?	
21	34

Word problems:

In year 3, there are 21 children and
year 4, there are 34 children.
How many children in total?

$21 + 34 = 55$. Prove it

21

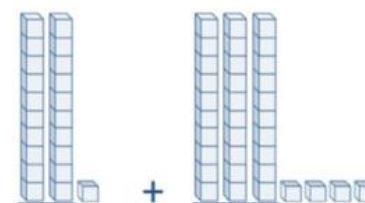
+34

$21 + 34 =$



$= 21 + 34$

Calculate the sum of twenty-or
and thirty-four.



Missing digit problems:

10s	1s
	?
?	5

Subtraction - Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

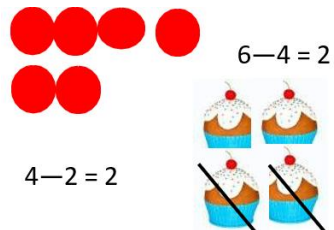
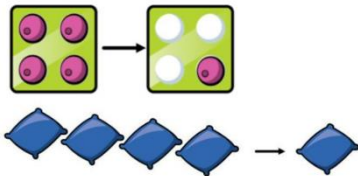
Year 1

Concrete

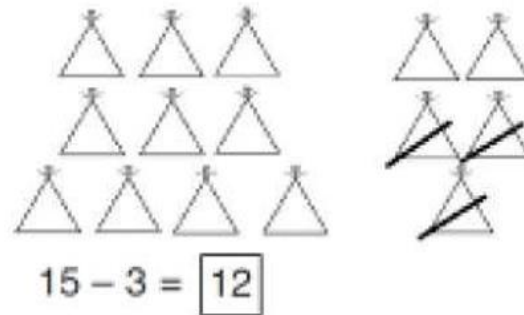
Take away ones.

Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).

$$4 - 3 = 1$$



Pictorial



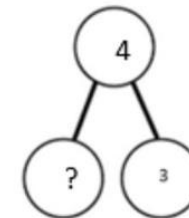
Cross out drawn objects to show what has been taken away.

Abstract

$$4 - 3 =$$

$$\boxed{} = 4 - 3$$

4	
3	?



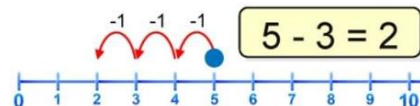
$$7 - 4 = 3$$

$$16 - 9 = 7$$

Counting back.

Counting back (using number lines or number tracks) children start with 6 and count back 2.

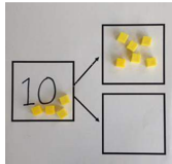
$$6 - 2 = 4$$



Count back in ones using a number line.

Put 13 in your head, count back 4. What number are you at?

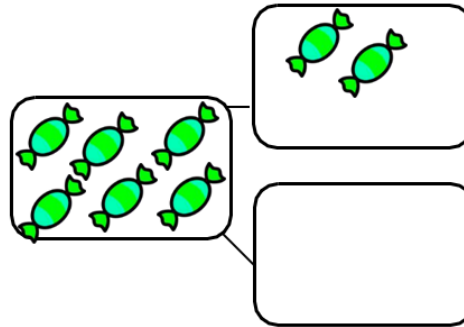
Related subtraction facts within 20. PW Model.



Link to addition. Use PPW model to model the inverse.

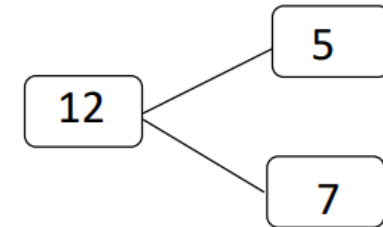
If 10 is the whole and 6 is one of the parts, what is the other part?

$$10 - 6 = 4$$



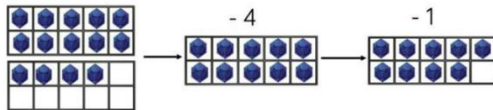
Use pictorial representations to show the part.

Move to using numbers within the part whole model.

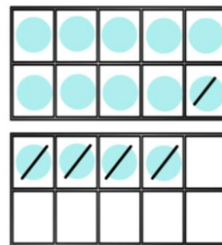


Make 10

Making 10 using ten frames.
14 - 5



Children to present the ten frame pictorially and discuss what they did to make 10.



$$16 - 8$$

How many do we take off first to get to 10? How many left to take off?

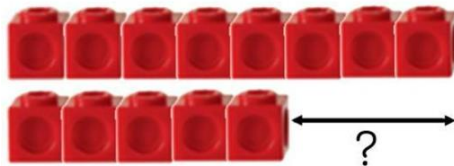
Year 2

Concrete

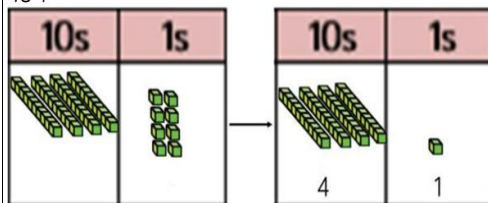
Finding the difference

Finding the difference (using cubes, Numicon or Cuis Rods etc.).

Calculate the difference between 8 and 5.

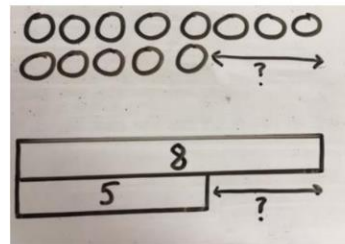


Column method using base 10.
48-7

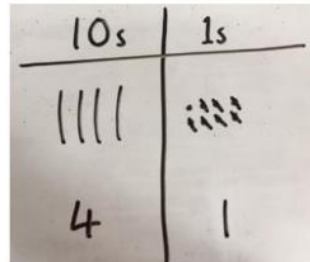


Pictorial

Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.



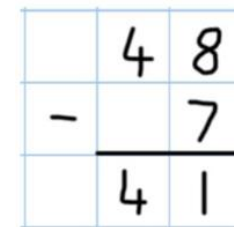
Children to represent the base 10 pictorially.



Abstract

Hannah has 12 sweets, and her sister has 5.
How many more does Hannah have than her sister?

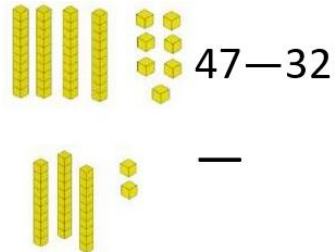
Column method or children could count back 7.



Year 3

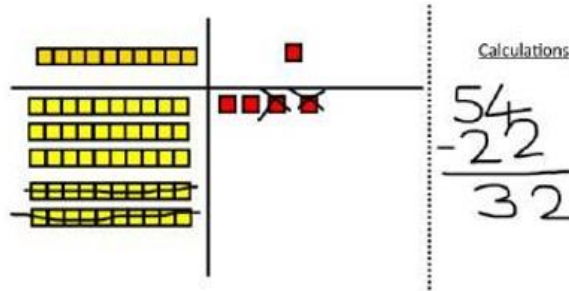
Concrete

Column subtraction without regrouping



Use base 10 or Numicon to model

Pictorial



Draw representations to support understanding.

Abstract

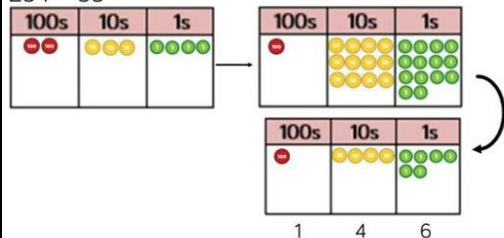
Calculations

$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

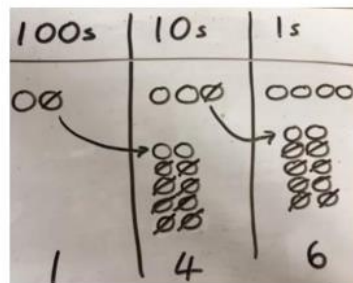
$$\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$$

Column method with regrouping.

Column method using place value counters.
234 - 88



Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} \overset{2}{2} \overset{1}{3} 4 \\ - 88 \\ \hline 6 \end{array}$$

Year 4

Concrete

As above using 4 digit numbers.
Model process of exchange using Numicon, base ten and then move to place value counters.

$$7,438 - 7,206 =$$

Th	H	T	O
			

Pictorial

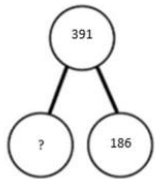
Children to draw place value counters and show their exchange
(See Year 3).

Abstract

	7	4	3	8
—	7	2	0	6
	0	2	3	2

$$\begin{array}{r} 7438 \\ - 7206 \\ \hline 232 \end{array}$$

Conceptual variation: different ways to ask children to solve 391 - 186



	391
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\square = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

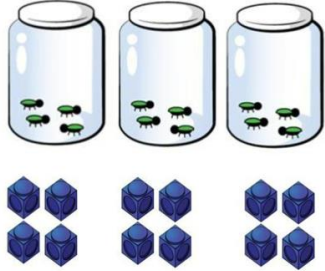
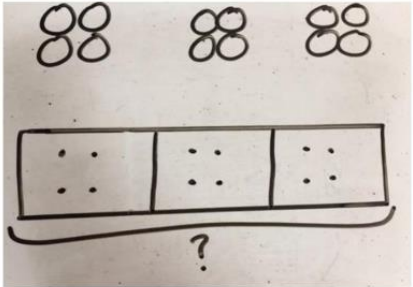
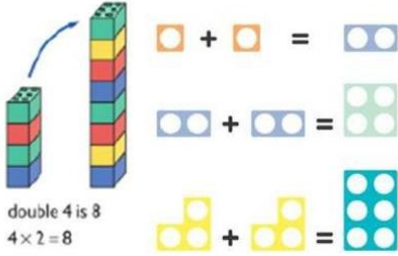
What is 186 less than 391?

Missing digit calculations

$$\begin{array}{r} 39\square \\ - \square\square6 \\ \hline \square05 \end{array}$$

Multiplication - Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

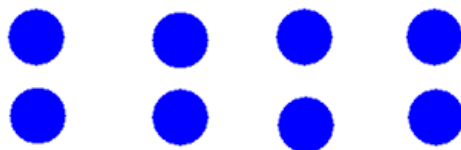
Year 1

Concrete	Pictorial	Abstract
Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group.  The image shows three jars, each containing four ants. Below the jars are three groups of four blue cubes, each group arranged in a 2x2 square.	Children to represent the practical resources in a picture and use a bar model.  The image shows three bags, each containing four sweets. Below the bags is a bar model divided into three equal sections, each containing four dots. A bracket underneath the bar model is labeled with a question mark. There are 4 sweets in 1 bag. How many are there in 3 bags?	$4 + 4 + 4 = 12$ $3 \times 4 = 12$ Write addition sentences to describe objects and pictures.  The image shows five pairs of boots, each pair consisting of a blue boot and a red boot. Below the boots is the equation $2 + 2 + 2 + 2 + 2 = 10$.
Doubling - children carry out practical activities using manipulatives including cubes and Numicon to demonstrate doubling.  The image shows a stack of four cubes being doubled to a stack of eight cubes. To the right are three Numicon shapes: two orange shapes (each with one dot) equaling one blue shape (with two dots), two blue shapes (each with two dots) equaling one green shape (with four dots), and two yellow shapes (each with four dots) equaling one teal shape (with eight dots). double 4 is 8 $4 \times 2 = 8$	Draw pictures to show how to double numbers. Double 4 is 8  The image shows two groups of four purple squares. The first group is arranged in a 2x2 square, and the second group is arranged in a 2x2 square.	Write double numbers. $4 + 4 = 8$ $5 + 5 = 10$

Counting in multiples. Count the groups as children skip count.



Children make representations to show counting in multiples.



Count in multiples aloud. Write sentences with multiple numbers.

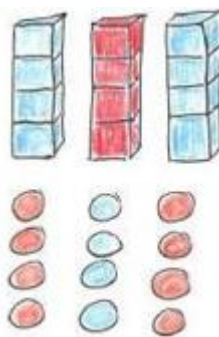
2,4,6,8,10

5,10,15,20,25,30

Make equal groups (Arrays) using manipulatives.



Draw arrays and make representations.

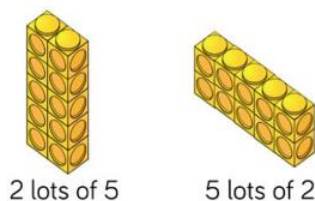


Use language such as:

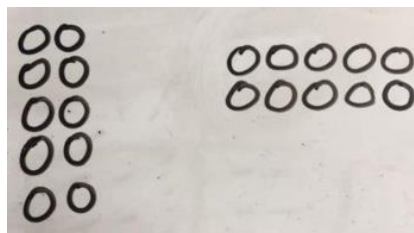
4 groups of 2 is 8

4 lots of 2 is 8

Use cube arrays to illustrate commutativity.



Children to represent arrays pictorially.

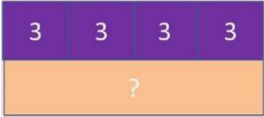
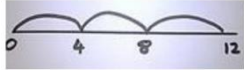
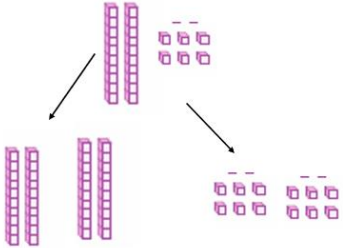
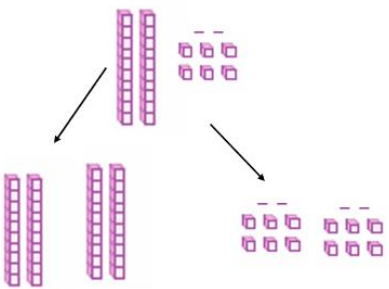
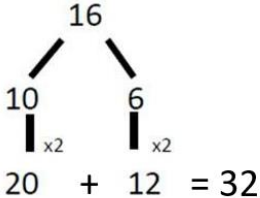
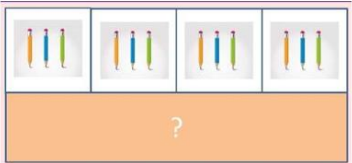
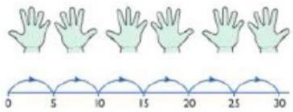
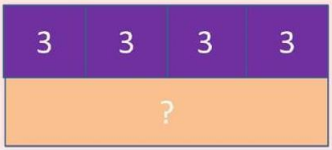


Children to be able to use arrays to write:

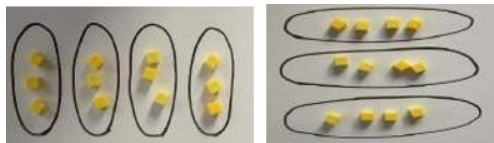
$2 + 2 + 2 + 2 + 2 = 10$

$5 + 5 = 10$

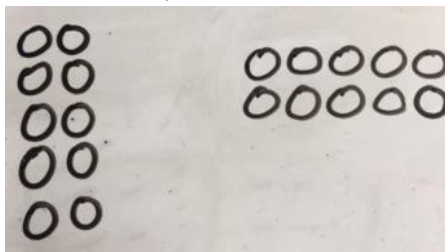
Year 2

Concrete	Pictorial	Abstract
Repeated addition - see Y1 examples.	 Draw bar model representations to show repeated addition.	$3 \times 4 = 12$ 
Doubling - Model doubling using dienes and place value counters.  $40 + 12 = 52$	Draw representations to show how to double numbers. 	Partition a number and then double each part before adding it back together.  $20 + 12 = 32$
Counting in multiples of 2,3,5 and 10. Count the groups as the children skip count.  $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$  	Number lines, counting sticks, bar models and 100 squares should be used to show representation of counting in multiples.   	Count aloud in multiples of a number. Write sentences with multiples of numbers. 0,2,4,6,8,10 0,3,6,9,12,15 0,5,10,15,20,25,30 0,10,20,30,40,50 $4 \times 2 = \boxed{}$

Explore commutativity by creating arrays using counters, cubes, and Numicon. Pupils should understand that the order of multiplication does not affect the answer.

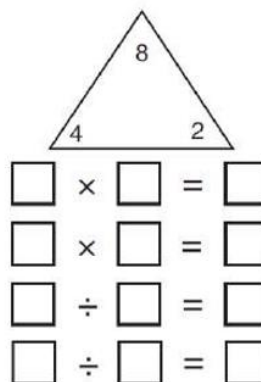


Use representations of arrays to show different calculations and explore commutativity.



$$\begin{aligned} 2 \times 5 &= 10 \\ 5 \times 2 &= 10 \\ 10 &= 2 \times 5 \\ 10 &= 5 \times 2 \end{aligned}$$

Explore the Inverse. Use cubes to explore the relationship between multiplication and division.



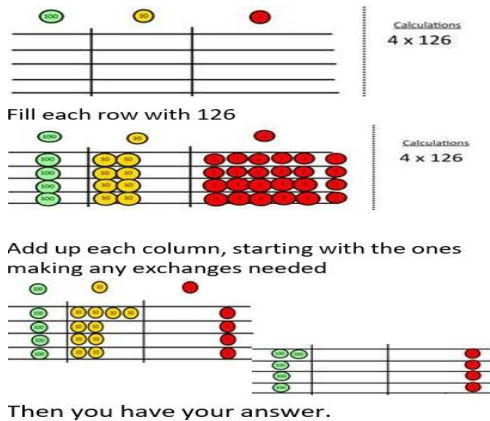
Fact families - children should be able to write and learn 4 associated facts.

$$\begin{aligned} 2 \times 4 &= 8 \\ 4 \times 2 &= 8 \\ 8 \div 2 &= 4 \\ 8 \div 4 &= 2 \end{aligned}$$

Year 3

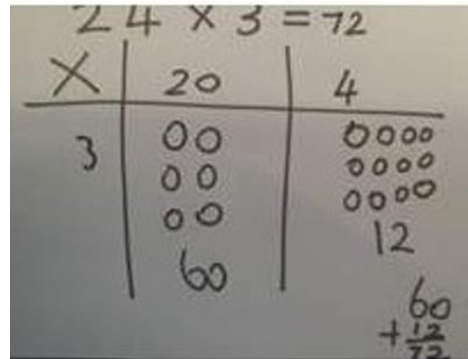
Concrete

Grid method - model using place value counters.



Pictorial

Children can represent their work by drawing the place value counters using colour to show the different amounts or use circles in the different columns. Bar models may also be used.



Abstract

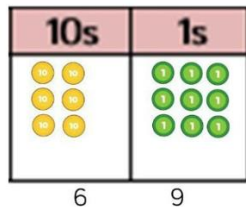
X	30	5
7	210	35

$$210 + 35 = 245$$

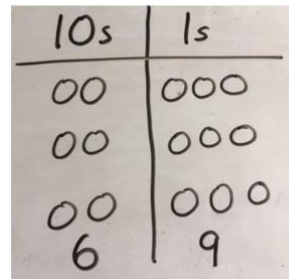
	10	8
10	100	80
3	30	24

Column multiplication.

Formal column method with place value counters (base 10 can also be used.) 3×23



Children to represent the counters pictorially.



Children to record what it is they are doing to show understanding.

$$3 \times 23 \quad 3 \times 20 = 60$$

$$20 \quad 3 \quad 3 \times 3 = 9$$

$$60 + 9 = 69$$

Move to formal column method when secure in above.

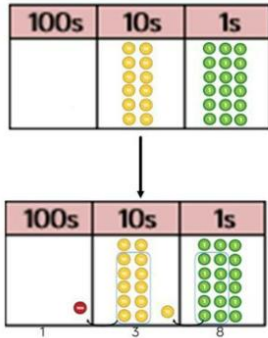
$$\begin{array}{r} 23 \\ \times 3 \\ \hline 69 \end{array}$$

Year 4

Concrete

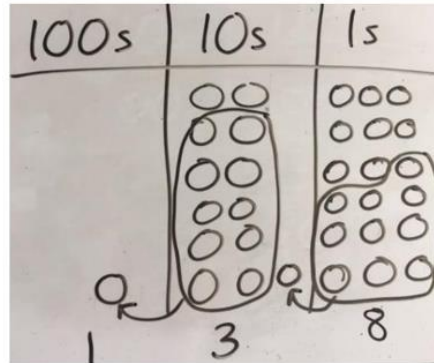
Recap methods from Year 3.

Formal column method with place value counters
6 x 23



Pictorial

Children to represent the counters/base 10, pictorially
e.g. the image below.

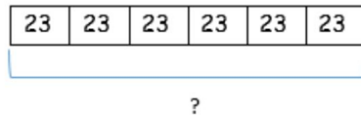


Abstract

Move to formal written method.

$$\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ \hline 1200 \\ 1308 \end{array}$$

Conceptual variation: different ways to ask children to solve 6 x 23



Lucy had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23

$$6 \times 23 =$$

$$\square = 6 \times 23$$

$$\begin{array}{r} 6 \quad 23 \\ \times 23 \\ \hline \end{array} \quad \begin{array}{r} 6 \quad 23 \\ \times 6 \\ \hline \end{array}$$

What is the calculation?
What is the product?

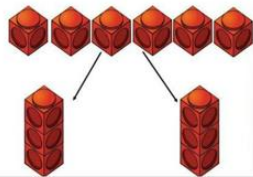


Division - Key language: share, group, divide, divided by, half

Year 1

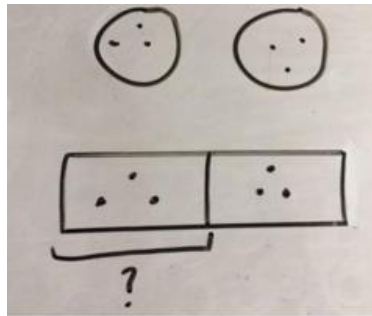
Concrete

Sharing using a range of objects.



Pictorial

Represent the sharing pictorially.



8 shared between 2 is 4

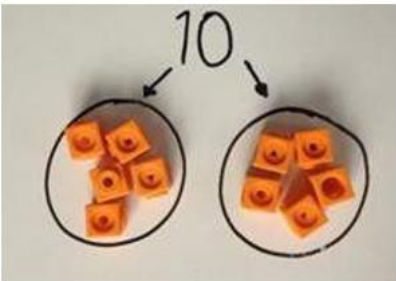
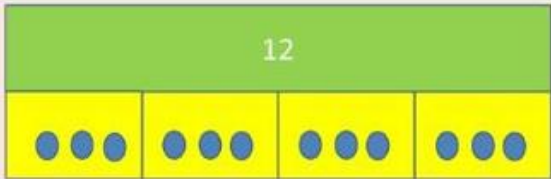
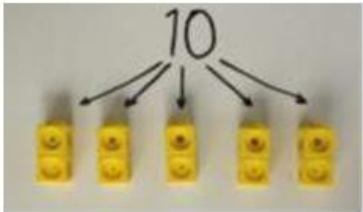
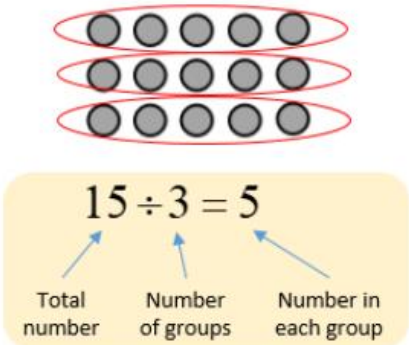
Abstract

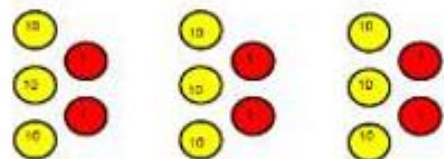
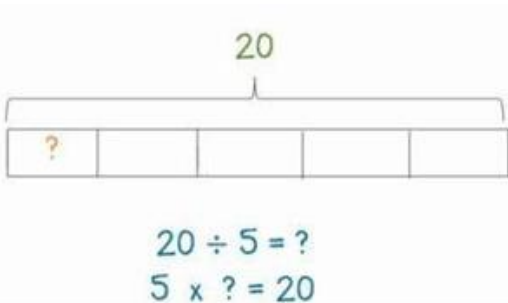
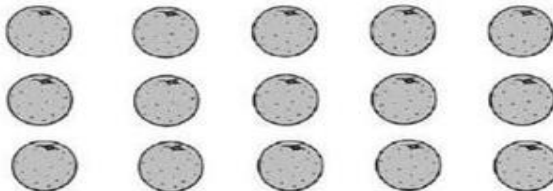
12 shared between 3 is 4.

8 shared between 2 is 4.

Introduce divide symbol when confident with language.

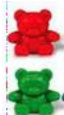
$$8 \div 2 = 4$$

Year 2		
Concrete	Pictorial	Abstract
Division as sharing  I have 10 cubes; can you share them equally in 2 groups?	Children use bar modeling or arrays to show and support understanding.  $12 \div 4 = 3$	$8 \div 2 = 4$ $12 \div 2 = 6$
Division as grouping. Divide quantities into equal groups. Use cubes, counters, objects, or place value counters to aid understanding. 	Children use bar modeling or arrays to show and support understanding. 	Divide 15 into 3 groups. How many are in each group? $15 \div 3 = 5$

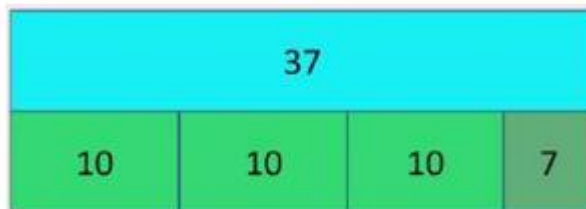
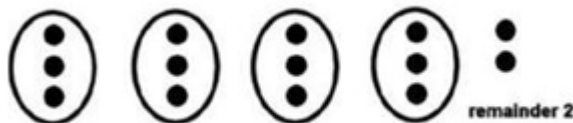
Year 3		
Concrete	Pictorial	Abstract
Division as grouping. $96 \div 3 = 32$ 	Use bar modelling to aid solving division problems. 	How many groups of 5 are in 20? $20 \div 5 = 4$
Division with arrays. (Inverse - link division and multiplication by creating an array and thinking about number sentences which can be created.) E.g., $15 \div 3 = 5$ $15 \div 5 = 3$ $5 \times 3 = 15$ $3 \times 5 = 15$ 	Draw an array and use lines to split the array into groups to make multiplication and division sentences 	Find the inverse of multiplication and division sentences by creating eight linking number sentences. $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Division with remainders.

Divide objects between groups to see how much is left over. E.g., $14 \div 3 = 4 \text{ r } 2$



Draw dots and group them to divide an amount and clearly show a remainder or use bar models to show remainders.



Complete written divisions showing remainder.

$$\begin{array}{ccccccc} 29 & \div & 8 & = & 3 & \text{REMAINDER } 5 \\ \uparrow & & \uparrow & & \uparrow & & \uparrow \\ \text{dividend} & & \text{divisor} & & \text{quotient} & & \text{remainder} \end{array}$$

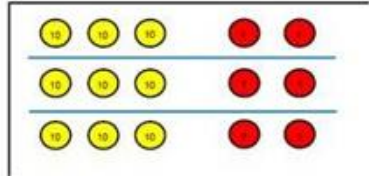
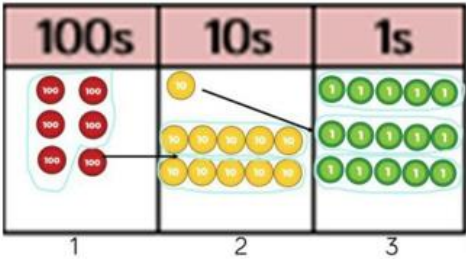
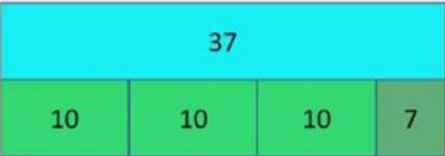
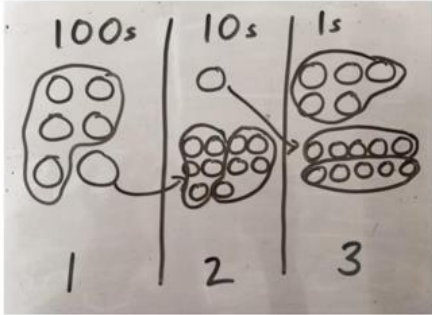
Move to short division when confident.
(With and without remainders.)

Begin with divisions that divide equally with no remainder.

$$\begin{array}{r} 218 \\ 4 \overline{) 872} \\ \underline{8} \\ 72 \\ \underline{72} \\ 0 \end{array}$$

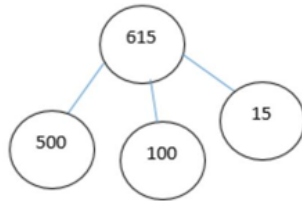
Move onto divisions with a remainder.

$$\begin{array}{r} 86 \text{ r } 2 \\ 5 \overline{) 432} \\ \underline{4} \\ 32 \\ \underline{30} \\ 2 \end{array}$$

Year 4		
Concrete	Pictorial	Abstract
$96 \div 3$ Tens Units 3 2  Use place value counters to divide using the bus stop method alongside. Short division using place value counters to group. $615 \div 5$ 	Children continue to draw arrays and bar models to help them divide numbers into equal groups/find remainders.   	Begin with divisions that divide equally with no remainder. $\begin{array}{r} 218 \\ 3 \overline{) 872} \end{array}$ Move onto divisions with a remainder. $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$ Increase to 4-digit numbers. Finally, move to divisions with decimals to divide accurately. $\begin{array}{r} 0663 \text{ r } 5 \\ 8 \overline{) 53029} \end{array}$ $\begin{array}{r} 14.6 \\ 16 \overline{) 511.0} \end{array}$

Conceptual variation: different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?
What is the answer?

100s	10s	1s